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THE NONPERTURBATIVE EQUATION FOR THE INFRARED $\Pi_{44}(0)$-LIMIT IN THE TEMPORAL AXIAL GAUGE [1]

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Abstract

The nonperturbative equation for the infrared $\Pi_{44}(0)$-limit is built by using the Slavnov-Taylor identity to define the three-gluon vertex function in the temporal axial gauge. We found that all vertex corrections should be taken into account along with the standard ring graphs to keep the gauge covariance throughout calculations and to give correctly the nonperturbative g^3-term. This term is explicitly calculated and compared with the previously known results.

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Recently the interest has been revived (see e.g. Refs.[1,2]) to calculate of the Debye mass beyond the leading term and to investigate of its gauge dependence. This problem is strongly connected with the analogous calculations of the infrared $\Pi_{44}(0)$-limit which are more simple but today they are reliably known only up to g^2-order. The next-to-leading term (the one of order g^3) was found many years ago (at first in paper [3] and then in other papers [4,5] and [6]) but till now its accuracy is not confirmed. Unfortunately these results are qualitatively different and show that the g^3-term found for the infrared Π_{44}-limit is a gauge-dependent quantity and its coefficient (that is more essential) is very sensitive to keeping the gauge covariance throughout calculations. The problem aggravates when the $\Pi_{44}(0, \mathbf{p})$-quantity is calculated but namely this limit (as it was shown in Ref.[1,2]) is needed to define the Debye screening. However there are many reasons at first to find reliably the infrared $\Pi_{44}(0)$-limit by using the temporal axial gauge since this gauge is singled out both for keeping the gauge covariance and for calculating the Debye screening.

The goal of this paper is to derive the nonperturbative equation for the infrared $\Pi_{44}(0)$-limit by operating the standard Green function technique within the temporal axial gauge. The obtained equation takes into account all perturbative graphs (the ring graphs as well as the vertex corrections) and its gauge covariance is guaranteed by exploiting the exact Slavnov-Taylor identities to find the nonperturbative three-gluon vertex. This vertex is qualitatively different from the bare one and the derived equation is free from any divergencies. The g^3-term is found to be a positive correction to the leading one and we compare it with the previously known results.

It is well-known that the temporal axial gauge is convenient for building the nonperturbative schemes since the choice of the gauge vector $\mathbf{n}_\mu$ to be parallel to the medium one $\mathbf{u}_\mu$ considerably simplifies the Green function technique. The exact polarization tensor (in the axial gauge) is determined by only two tensor structures [6]

$$\Pi_{\mu\nu}(k) = G\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + (F - G)B_{\mu\nu}\,, \tag{1}$$

and the gluon propagator has a rather simple form

$$\mathcal{D}_{ij}(k) = \frac{1}{k^2 + G}\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) + \frac{1}{k^2 + F}\frac{k^2}{k_4^2}\frac{k_i k_j}{\mathbf{k}^2}. \tag{2}$$

The scalar functions $F(k)$ and $G(k)$ are defined as follows

$$G(k) = \frac{1}{2}\left(\sum_i \Pi_{ii}(k) + \frac{k_4^2}{\mathbf{k}^2}\Pi_{44}(k)\right)\,, \qquad F(k) = \frac{k^2}{\mathbf{k}^2}\Pi_{44}(k)\,, \tag{3}$$

and they should be calculated through the graph (or another) representation for Π. Due to a peculiarity of the temporal axial gauge the functions $\mathcal{D}_{44}(k)$ and $\mathcal{D}_{4j}(k)$ are completely eliminated from the formalism but there is a very specific singularity ($k_4^2 = 0$) which requires a very delicate treatment. The exact Slavnov-Taylor identity for the three-gluon vertex function has a rather simple form

$$r_\mu \Gamma^{abc}_{\mu\nu\gamma}(r,\, p,\, q) = igf^{abc}[\mathcal{D}^{-1}_{\nu\gamma}(p) - \mathcal{D}^{-1}_{\nu\gamma}(q)]\,, \tag{4}$$

and namely this fact is a doubtless advantage of the axial gauge. Eq.(4) is our main instrument and we exploit it to build the nonperturbative vertex function for calculating the infrared $\Pi_{44}(0)$-limit. We also use Eq.(4) in its differential form which allows in many cases (see e.g. Ref.[7]) to define the exact infrared limit of the three-gluon vertex in a very convenient manner. For example, the infrared $\Gamma^{abc}_{4ij}(-p, 0, p)$-limit can be easily found from the standard identity

$$\Gamma^{abc}_{4ij}(-p,\, 0,\, p) = -igf^{abc}\frac{\partial \mathcal{D}^{-1}_{4j}(p)}{\partial p_i}\,, \tag{5}$$

which directly results from Eq.(4). This limit being exact has a rather simple form

$$\Gamma^{abc}_{4ij}(-p,\, 0,\, p) = igf^{abc}\left\{\delta_{ij}[1 + \frac{F(p)}{p^2}] + p_j\frac{\partial}{\partial p_i}\left[\frac{F(p)}{p^2}\right]\right\} p_4\,, \tag{6}$$

and depends on one function which determines the usual representation for the $\mathcal{D}^{-1}_{4i}(p)$-propagator

$$\mathcal{D}^{-1}_{4j}(p) = -\left(1 + \frac{F(p)}{p^2}\right) p_j p_4. \tag{7}$$

Unfortunately the limit (6) is not our case but we shall return back to Eq.(6) when the nonperturbative expression for the infrared $\Gamma^{abc}_{4ij}(0, p, -p)$-limit is discussed.

The exact graph representation for the gluon polarization tensor is well-known (see e.g. papers [6,8]) and contains (in the axial gauge) the standard four nonperturbative graphs. However if one considers the Π_{44}-components only two one-loop nonperturbative graphs are essential since the rest graphs (the two very complicated ones) are equal to zero exactly. The analytical expression for the first two graphs has a rather simple form and after some algebra being performed (by taking into account that $\Gamma^{abc}_{ij4} = -igf^{abc}\Gamma_{ij4}$) the equation for m^2_E (where $m^2_E = \Pi_{44}(0)$) is found to be

$$
\begin{aligned}
m^2_E &= \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \mathcal{D}_{ii}(p) \\
&- \frac{g^2 N}{2\beta} \sum_{p_4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} 2p_4 \left[\mathcal{D}_{li}(p)\Gamma_{ij4}(p,\, -p,\, 0)\mathcal{D}_{jl}(p)\right] ,
\end{aligned}
\tag{8}
$$

and we are going to solve it keeping the gauge covariance at each step of the calculations. Here all functions (including the vertex one) are exact and our main problem is to find the nonperturbative expression for the infrared $\Gamma_{ij4}(p, -p, 0)$-limit.

Unfortunately the exact expression for the infrared $\Gamma_{ij4}(p, -p, 0)$-limit (which is not Eq.(6)) lies beyond our possibilities and therefore only its nonperturbative ansatz will be represented by following the more general formula obtained in Ref.[7]. This formula is found to be

$$
\begin{aligned}
\Gamma^{abc}_{4ij}(q,\, r,\, p) = -igf^{abc} \Bigg\{ & \delta_{ij}(r_4 - p_4) - \frac{1}{r^2 - p^2} \left[\left(\frac{G(r)}{\mathbf{r}^2} - \frac{F(r)}{r^2} \frac{r_4^2}{\mathbf{r}^2} \right) \right. \\
& \left. - \left(\frac{G(p)}{\mathbf{p}^2} - \frac{F(p)}{p^2} \frac{p_4^2}{\mathbf{p}^2} \right) \right] [(\mathbf{pr})\delta_{ij} - p_i r_j](r_4 - p_4) \\
& + \delta_{ij} \left(r_4 \frac{F(r)}{r^2} - \frac{F(p)}{p^2} p_4 \right) + \frac{1}{q^2 - r^2} \left(\frac{F(q)}{q^2} - \frac{F(r)}{r^2} \right) q_i r_4 (q - r)_j \\
& + \frac{1}{p^2 - q^2} \left(\frac{F(p)}{p^2} - \frac{F(q)}{q^2} \right) (p - q)_i p_4 q_j \\
& - \frac{1}{r^2 - p^2} \left(\frac{F(r)}{r^2} - \frac{F(p)}{p^2} \right) r_4 p_4 (r - p)_4 \delta_{ij} \Bigg\} ,
\end{aligned}
\tag{9}
$$

and it is valid for any momentum set including the soft domain. To find

Eq.(9) the standard inverse gluon propagator is used

$$\mathcal{D}_{ij}^{-1}(p) = \left(\delta_{il} - \frac{p_i p_j}{\mathbf{p}^2}\right)(p^2 + G(p)) + \left(1 + \frac{F(p)}{p^2}\right) p_4^2 \frac{p_i p_j}{\mathbf{p}^2} \,, \qquad (10)$$

and the exact Slavnov-Taylor identities were exploited (see Ref.[7] for details). The transversal part of the $\Gamma_{4ij}^{abc}(q, r, p)$- function is omitted from Eq.(9) since it is not essential for what follows. Of course, it is necessary to bear in mind to exclude all singularities from Eq.(9) for any momentum going to zero.

The vertex function (9) easily reproduces the exact formula (6) if one momentum goes to zero (at first $r_4 = 0$ and then $|\mathbf{r}| \to 0$) but it is more essential that one can exploit this representation in a more general case to find the infrared $\Gamma_{ij4}^{abc}(p, -p, 0)$-limit. The final result has the form

$$\begin{aligned}\Gamma_{ij4}^{abc}(p,\, -p,\, 0) = -igf^{abc} \Bigg\{ & 2\delta_{ij}\left(1 + \frac{F(p)}{p^2}\right) \\ & + 2\frac{p_4^2}{\mathbf{p}^2}\left(\frac{F(p)}{p^2}\right)\left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2}\right) + \frac{1}{|\mathbf{p}|}\left[\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{G(p)}{\mathbf{p}^2}\right)\right][\mathbf{p}^2\delta_{ij} - p_i p_j] \\ & + p_4^2\left[\frac{1}{|\mathbf{p}|}\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{F(p)}{p^2}\right)\right]\frac{p_i p_j}{\mathbf{p}^2}\Bigg\} p_4 \,, \end{aligned} \qquad (11)$$

and we apply this representation for treating Eq.(8). The vertex found is qualitatevly different from the bare one and we hope that Eq.(11) will be usefull for many applications.

All algebra within Eq.(8) is very simple and therefore omitted. The equation for m_E^2 is found as follows

$$\begin{aligned} m_E^2 = -\frac{g^2 N}{\beta}\sum_{p_4}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Bigg\{ & \frac{1}{p_4^2}\frac{1}{1 + \frac{F(p)}{p^2}} - \frac{2}{p^2 + G(p)} \\ & + \frac{4p_4^2}{[p^2 + G(p)]^2} + \frac{1}{|\mathbf{p}|}\left[\frac{\partial}{\partial|\mathbf{p}|}\left(\frac{F(p)}{p^2}\right)\right]\left[1 + \frac{F(p)}{p^2}\right]^{-2} \\ & + \left[\frac{2p_4^2}{|\mathbf{p}|}\left(\frac{\partial G(p)}{\partial|p|}\right) - 4p_4^2\left(\frac{G(p)}{\mathbf{p}^2} - \frac{F(p)}{\mathbf{p}^2}\right)\right]\frac{1}{[p^2 + G(p)]^2}\Bigg\} \,, \end{aligned} \qquad (12)$$

and it is the main subject of the following discussion. Eq.(12) correctly summarizes not only the ring graphs but essentially exploits the dressed

three-gluon vertex in a nonperturbative manner and it is very probably that this equation is exact for the g^3-term. Of course, Eq.(12) correctly reproduces the leading order for m_E^2 if the functions G(p) and F(p) are omitted

$$m_E^2 = \frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\partial}{\partial p_4} \left[\frac{2p_4}{p^2} + \frac{1}{p_4} \right] , \tag{13}$$

and the standard regularization for the temporal axial gauge is used

$$\frac{1}{\beta} \sum_{p_4} \frac{1}{p_4^2} = 0. \tag{14}$$

No other terms should be taken into account in Eq.(12) since the rest graphs (which usually determine the $\Pi_{\mu\nu}$-tensor) are exactly equal to zero if the Π_{44}-quantity is only considered. This fact is due to a simple Lorentz tensor structure of the bare Γ_4-vertex function and the specific feature of the temporal axial gauge where the $\mathcal{D}_{44}$-function is eliminated from the formalism. The g^3-term (as well as all the other ones) should be completely determined by Eq.(12) and we found that this equation is free from any divergencies (of course if the full sum over p_4 is taken into account).

Our next task is to find the appropriate equation for calculating the g^3-term on the basis of Eq.(12). This is a pure nonperturbative term and arises within Eq.(12) when the soft momentum region is exploited. Due to the different infrared behaviour of the functions G(p) and F(p) only the latter gives the appropriate contribution to reproduce the g^3-term and all other terms being of g^4-order can be omitted. The final equation for the g^3-term (here the δm_E^2-term) has a rather simple form

$$\begin{aligned} \delta m_E^2 \;\; = \;\; & -\frac{g^2 N}{\beta} \sum_{p_4} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left\{ \frac{1}{p_4^2} \frac{1}{1 + \frac{F(p)}{p^2}} \right. \\ & \left. + \frac{1}{|\mathbf{p}|} \left[\frac{\partial}{\partial |\mathbf{p}|} \left(\frac{F(p)}{p^2} \right) \right] \left[1 + \frac{F(p)}{p^2} \right]^{-2} \right\} , \end{aligned} \tag{15}$$

and it can be solved independently from Eq.(12). However, there is a question with the first term in Eq.(15) which contains the specific singularity of the temporal axial gauge and its analytical behaviour is not clear. Nevertheless

we insist that this term is equal to zero if only the static $\Pi_{44}(0)$-limit is used

$$\Pi_{44}^{(2)}(p_4 = 0, |\mathbf{p}| \to 0) = \frac{g^2 N}{3\beta^2}\,, \tag{16}$$

and all calculations are performed in the standard infrared manner (when the sum over p_4 is replaced by one term with $p_4 = 0$). Finally the above equation is found to be

$$\delta m_E^2 = -\frac{g^2 N}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{|\mathbf{p}|} \left[\frac{\partial}{\partial |\mathbf{p}|} \left(\frac{\Pi_{44}^{(2)}(0)}{\mathbf{p}^2} \right) \right] \left[1 + \frac{\Pi_{44}^{(2)}(0)}{\mathbf{p}^2} \right]^{-2}\,, \tag{17}$$

where all functions are known and the integral is calculated in the usual manner. Our result has a rather simple form

$$m_E^2 = [\frac{g^2 N}{3} + \frac{3}{4\pi}(\frac{g^2 N}{3})^{3/2}]T^2\,, \tag{18}$$

and it is in an agreement with the results [1,2] and [5] if those are considered in the Feyman gauge only. The other results (see Ref.[3,4]) should be checked to solve reliably the question of the gauge dependence of the g^3-term.

In conclusion we note that the result found for the infrared $\Pi_{44}(0)$-limit is not reproduced through the effective action calculated in a constant background field (see e.g. Refs.[9,10]). This is the case only for the leading term but in a general case the correspondence seems to be more complicated. This fact needs further investigations to establish finally the status of the g^3-term and its connection with the Debye screening. However there is a hope that the latter problem can be solved independently from the scenario with the magnetic mass and the question with the magnetic screening is separated (at least in the temporal axial gauge) into the outstanding problem. There is also a nonperturbative equation [11]

$$\begin{aligned} m_M^2 \;&=\; \frac{3N^2 g^4}{4\beta^2} \sum_{p_4,q_4,r_4} (2\pi)^3 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{d^3\mathbf{r}}{(2\pi)^3} \delta^{(4)}(p+q+r) \times \\ &\times\; \mathcal{D}_{jn}(q)\mathcal{D}_{it}(r) \frac{\partial}{\partial p_i} [\mathcal{D}_{nm}(p)\Gamma_{mjt}(-p,-q,-r)] \end{aligned} \tag{19}$$

but its solution which encounters the infrared divergencies is still unconfirmed. It is important that Eq.(19) is also accessible only in the temporal axial gauge and it results from the two rest nonperturbative graphs which are equal to zero in the above case.

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